



Lawrence High School's  
AP BC Calculus  
Required Summer Assignment

$$\int_a^b f'(x) dx = f(b) - f(a)$$

$$\frac{d}{dx} \int_a^x f(t) dt = f(x)$$



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Hello Incoming AP Calculus BC Students:

To be best prepared for AP Calculus BC in September, you are assigned a mandatory Summer Assignment, which is a review of some Precalculus topics and some Calculus topics. It is to be done NEATLY and on a SEPARATE sheet of paper. Points will be awarded only if the correct work is shown, and that work leads to the correct answer. Your completed packet is due in early September - the first few days of school. An AP BC Calculus Google Classroom has been set up for you.

**Google Classroom Group Code:**

- Go to [www.classroom.google.com](http://www.classroom.google.com)
- Click on I'm a Student
- Enter access code `css8w6e` to access the class.

This Google group has been established in order to provide you, the students, with support as a group. If you are struggling with concepts/material, there is a blog option to post questions to one another. In addition, you have the educational resources listed below for additional assistance. Remember the math course from the current school year is the prerequisite course for the course you have enrolled into for the Fall. Your personal notebook and handouts from this year's class is a resource that is at your disposal. The instructor might monitor the Google Classroom throughout the summer.

**Directions:**

- Complete ALL problems.
- Show all work for every problem on a separate piece of paper, with your name on each page.
- Your work should be neatly organized and clearly labeled.

**Grading:**

- Total of 158 points. Each question for #1-45 is worth 2 point. #46-70 individually marked.
- Any problem with no work shown will receive 0 points.
- When asked to explain or justify, you must write in full sentences and try not to use any pronouns ie: instead of saying "it has a zero," say "the function has a zero."
- Your teacher will enter your earned grade for Summer Assignment grade into Power School.



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## Resources:

For additional examples and support you can reference any of the sites listed below and search the skill/concept.

- KhanAcademy.com
- YouTube.com or TeacherTube.com (Reference Patrickjmt)
- MathIsPower4u.com
- IXL.com

DUE DATE: MONDAY, SEPTEMBER 9, 2019.

I look forward to seeing you in September!!!!

Mrs. Bradley

**A calculator may not be used on this part of the assignment. Show all work.**

1. If  $f(x) = \frac{\ln x}{x}$ , what is  $f'(x)$ ? 1) \_\_\_\_\_

2. How many points of inflection exist for the function  
 $y = \sin 2x$  on the open interval  $0 < x < \frac{\pi}{2}$ ? 2) \_\_\_\_\_

3.) If  $\frac{dy}{dx} = 2xy$ , then a possible solution for  $y$  is 3) \_\_\_\_\_

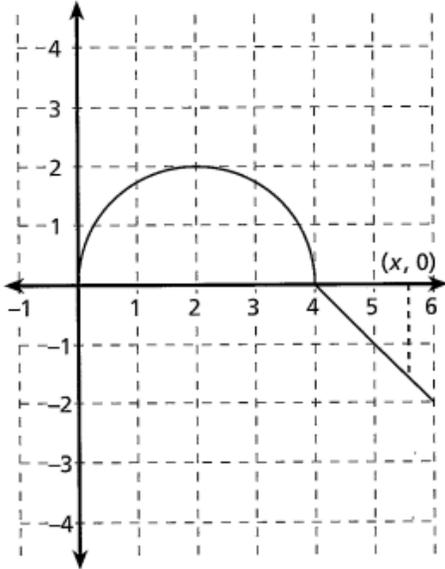
4) What is the equation of the line normal to the curve  
 $y = e^{2x} \ln(x)$  where  $x = 1$ ? 4) \_\_\_\_\_



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- 5) The graph of  $f(t)$  (shown below) consists of a semicircle and a line segment as shown to the right. Which of the following represents the value of  $\int_2^x f(t)dt$ , where  $x > 4$ ?



5) \_\_\_\_\_

- 6) What is the value of  $\lim_{x \rightarrow 0} \frac{\tan x}{x}$ ?

6) \_\_\_\_\_

- 7) For  $xy^2 - 3x = y^3$ , find  $y'$  when  $y = -1$

7) \_\_\_\_\_

- 8)  $\int 9xe^{3x^2} dx =$

8) \_\_\_\_\_

- 9) Given the piecewise function

$$f(x) = \begin{cases} 4 - bx^2, & -1 < x \leq 2 \\ abx, & 2 < x < 4 \end{cases}$$

and  $b$  as nonzero constants, what are all possible values of  $b$  that will make  $f(x)$  continuous and differentiable?

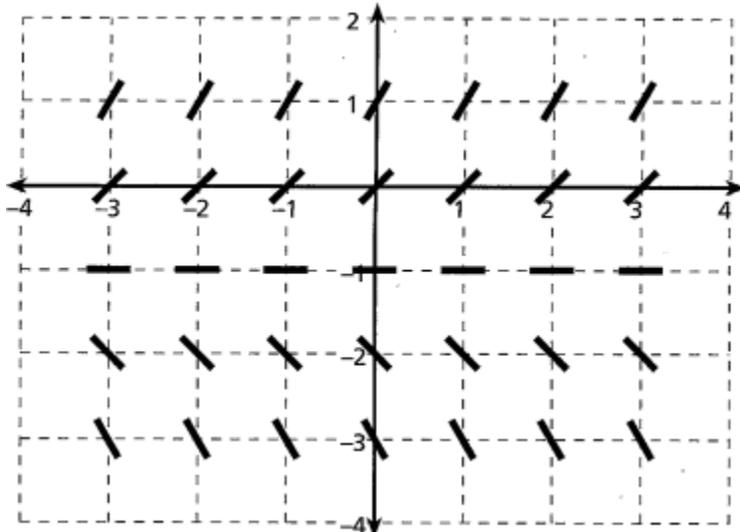
9) \_\_\_\_\_

- 10) The position of a particle moving in a line is  $s(t) = t^3 - 5t^2 + 2t - 13$ . What is the speed of the particle at  $t=2$ ?

10) \_\_\_\_\_



11) Which of the following differential equations has the slope field shown below?



11) \_\_\_\_\_

12)  $\int_1^6 \sqrt{x+3} dx =$

12) \_\_\_\_\_

13) The derivative of  $f(x)$  is given by

$$f'(x) = \frac{3x^2(x-2)(x+5)^{1/3}}{(x-6)}$$

$f(x)$  decreasing?

13) \_\_\_\_\_

14) The sum of two positive numbers is 5. What is the value of the larger number if the product of the smaller number and the cube of the larger is a maximum?

14) \_\_\_\_\_

15) If  $f(x) = \ln x$  and  $\frac{f'(x)}{-[f(x)]^2} = g'(x)$ , which of the following could be  $g(x)$ ?

15) \_\_\_\_\_



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- 16) For  $f(x) = \frac{2x-4}{x^2-x-2}$ , which of the following is true?  
I.  $f(x)$  has no relative extrema.  
II. There are vertical asymptotes at  $x=2$  and  $x=-1$ .  
III. There is a horizontal asymptote at  $y=0$ . 16) \_\_\_\_\_
- 17) Let  $F(x) = \int_1^x \cos(\pi t^2) dt$ . What is the value of  $F'(2)$ ? 17) \_\_\_\_\_
- 18) The line  $y = ax + k$  is tangent to the circle  $x^2 + (y-4)^2 = 20$  at the point  $(4, 6)$ . What is the value of  $(a+k)$ ? 18) \_\_\_\_\_
- 19) If  $\ln y = x^2 \ln x$ , what is  $\frac{dy}{dx}$  in terms of  $x$  and  $y$ ? 19) \_\_\_\_\_
- 20) If the derivative of a function is given as  $f'(x) = \frac{x-6}{e^x}$ , then in which open interval is the function both increasing and concave up? 20) \_\_\_\_\_
- 21) A particle moves along the  $y$ -axis so that its position at time  $t$  is given by  $y(t) = -3t^4 + 18t^2$ , for  $t \geq 0$ . What is the value of the velocity when the acceleration is 0? 21) \_\_\_\_\_
- 22) Which of the following expressions represents the average value of  $f(x) = \sqrt{2x-1}$  in  $[1, 3]$ ? 22) \_\_\_\_\_



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23) If  $f(x) = \sin x$ ,  $g(x) = \cos(2x)$ , and  $h(x) = f(g(x))$ ,

What is  $h' \left( \frac{\pi}{4} \right)$ ?

23) \_\_\_\_\_

24) A young girl, 5 feet tall, is walking away from a lamppost which is 12 feet tall. She walks at a constant rate of 2 feet per second and notices that, as she moves away from the lamppost, the length of her shadow is increasing. How fast is the length of her shadow increasing in feet per second when she is 20 feet from the post?

24) \_\_\_\_\_

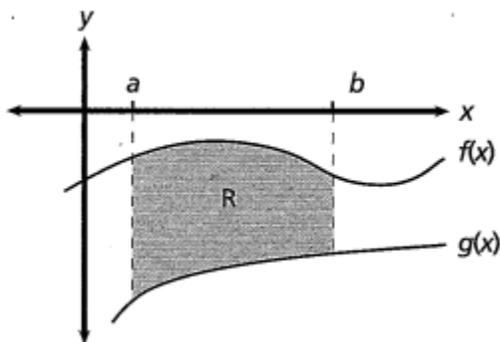
25) If  $f(x) = 3x^3 + 5x$  and  $g(x) = f^{-1}(x)$ , what is  $g'(8)$ ?

25) \_\_\_\_\_

26) Find  $\int \frac{x^3 - x^2 + 1}{x} dx$ .

26) \_\_\_\_\_

Use this graph of  $f(x)$  and  $g(x)$  for problems 27 and 28.





27) The graphs of  $f(x)$  and  $g(x)$  are shown above. If  $R$  is the region bounded by  $f(x)$ ,  $g(x)$ ,  $x = a$ , and  $x = b$ , which of the following represents the area of region  $R$ ?

I.  $\int_a^b [f(x) - g(x)] dx$

II.  $\int_b^a [f(x) g(x)] dx$

III.  $\int_b^a [g(x) - f(x)] dx$

27) \_\_\_\_\_

28. The graphs of  $f(x)$  and  $g(x)$  are shown in the figure above. If  $R$  is the region bounded by  $f(x)$ ,  $g(x)$ ,  $x = a$ ,  $x = b$ , set up volume when  $R$  is rotated about the line  $y = 1$ ?

28) \_\_\_\_\_

**A calculator may be used on this part of the assignment. Show the set up.**

29) The curve  $f(x) = \frac{\sec x}{x+2}$  passes through the point  $(0, \frac{1}{2})$ .

Use the equation of the tangent line to the curve at the point  $(0, \frac{1}{2})$  to approximate  $f(0.1)$ .

29) \_\_\_\_\_

30) At what value of  $x$  in the open interval  $-1 < x < 0$  will the instantaneous rate of change of the function  $f(x) = \tan^{-1}(x)$  equal the average rate of change of the function with respect to  $x$  on the closed interval  $-1 \leq x \leq 0$ ?

30) \_\_\_\_\_

31) If  $k > 1$ , the area under the curve  $y = kx^2$  from  $x=0$  to  $x=k$  is

31) \_\_\_\_\_



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Use the following information for problems 32 and 33.

The rate of natural gas sales for the year 1993 at a certain gas company is given by  $P(t) = t^2 - 400t + 160,000$ , where  $P(t)$  is measured in gallons per day and  $t$  is the number of days in 1993 (from day 0 to day 365).

32) To the nearest gallon, what is the total number of gallons of natural gas sales at this company for the 31 days (day 0 to day 31) of January 1993? 32) \_\_\_\_\_

33) To the nearest gallon, what is the average rate of natural gas sales at this company for the 31 days (day 0 to day 31) of January 1993? 33) \_\_\_\_\_

34) If  $f(-x) = -f(x)$  and  $f(2) = 3$  and  $f'(2) = \frac{1}{5}$ , then what is the equation of the tangent line to  $f(x)$  at  $x = -2$ ? 34) \_\_\_\_\_

35) A solid is generated by revolving the region bounded by the  $x$ -axis, the line  $x=5$ , and the function  $y = \ln x$  around the  $x$ -axis. What is the volume of the solid? 35) \_\_\_\_\_



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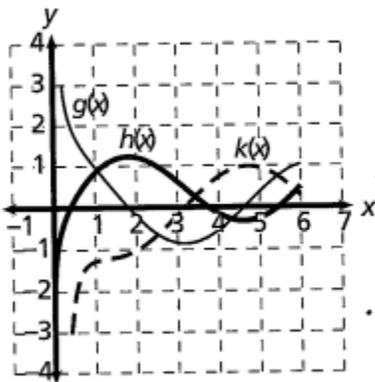
36) A continuous function  $g(t)$  is defined in the closed interval  $[0, 6]$  with values given in the table below. Using a midpoint Riemann sum with three subintervals of equal length, what is the approximate value of  $\int_0^6 g(t) dt$  is

$t$	$g(t)$
0	4
1	7
2	8
3	12
4	15
5	22
6	26

36) \_\_\_\_\_

37) A particle, initially at rest, moves in a line so that its acceleration is  $a(t) = \frac{10}{t+1}$  for  $t \geq 0$ . What is the velocity of the particle at time  $t = 4$ ?

37) \_\_\_\_\_





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- 38) Three functions  $g(x)$ ,  $h(x)$  and  $k(x)$  as graphed above represent a function  $f(x)$ , and its first and second derivatives,  $f'(x)$  and  $f''(x)$ , but not necessarily in this order. Using  $g(x)$ ,  $h(x)$  and  $k(x)$ , which ordered triple represents  $(f(x), f'(x), f''(x))$ ? 38) \_\_\_\_\_
- (A)  $[g(x), h(x), k(x)]$   
(B)  $[k(x), h(x), g(x)]$   
(C)  $[g(x), k(x), h(x)]$   
(D)  $[h(x), g(x), k(x)]$   
(E) none of these
- 39)  $\int \cos(7t + 3) dt =$  39) \_\_\_\_\_
- 40)  $\int_3^{10} 4f(x) dx = 20$  and  $\int_7^{10} 3f(x) dx = -12$ , then  $\int_3^7 11f(x) dx =$  40) \_\_\_\_\_
- 41) Which value of  $x$  best approximates the value which satisfies the Mean Value Theorem for the function  $f(x) = x^{\frac{2}{3}} + 1$  on the interval  $1 \leq x \leq 8$ ? 41) \_\_\_\_\_
- 42) What is  $g'(0)$  if  $g(x) = \frac{x-f(x)}{3-x^2}$ ,  $f(0)=1$ , and  $f'(0)=-1$  42) \_\_\_\_\_
- 43) For  $f(x) = \sec 2x$  and  $g(x) = \tan x$ , at what value of  $x$  on the interval  $\left(\frac{-\pi}{4}, \frac{\pi}{4}\right)$  do the graphs of  $f$  and  $g$  have tangent lines that are parallel? 43) \_\_\_\_\_



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44) For  $f(x) = ax^2 + bx^3$  what is the ordered pair  $(a, b)$  if the point  $(-1, 2)$  is an extrema of  $f(x)$ ? 44)\_\_\_\_\_

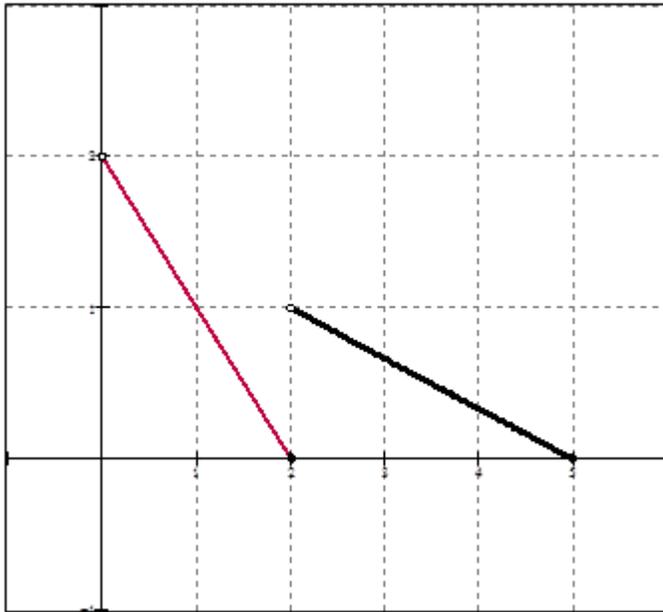
45) Find the volume of the solid whose base is bounded in the  $xy$ -plane by the graphs  $f(x) = \sqrt[3]{x+1}$  and  $g(x) = \frac{x}{3} - \frac{1}{3}$ , and whose cross sections taken perpendicular to the  $y$ -axis are isosceles right triangles with one leg of the right triangle in the  $xy$ -plane. 45)\_\_\_\_\_

**For the remainder of the packet use your discretion as to whether you should use a calculator or not. When in doubt, think about if I would use one.**

46) For what value of  $k$  are the two lines  $2x + ky = 3$  and  $x + y = 1$  (a) parallel? (b) perpendicular? 46)\_\_\_\_\_ (2)

47)\_\_\_\_\_ (2)

47) Write a piecewise formula for the function shown. Include the domain of each piece! (2)



Find the limits, if they exist. For #48-50 2 pts each

$$48) \lim_{x \rightarrow 9} \frac{\sqrt{x} - 3}{9 - x}$$

$$49) \lim_{h \rightarrow 0} \frac{\tan^{-1}(1+h) - \frac{\pi}{4}}{h} =$$

$$50) \lim_{h \rightarrow 0} \frac{\sec(\pi + h) - \sec(\pi)}{h} =$$

51) Let  $f$  be the function defined by  $f(x) = \frac{x + \sin x}{\cos x}$  for  $-\frac{\pi}{2} < x < \frac{\pi}{2}$ .

a) State whether  $f$  is an even function or an odd function. Justify answer. (2)

b) Find  $f'(x)$ . (2)

c) Write an equation for the line tangent to the graph of  $f$  at the point  $(0, f(0))$ .  
(2pts)



For #52-54, explain why each function is discontinuous and determine if the discontinuity is removable or nonremovable. 2 pts each

52)  $g(x) = \begin{cases} 2x-3, & x < 3 \\ -x+5, & x \geq 3 \end{cases}$

53)  $b(x) = \frac{x(3x+1)}{3x^2-5x-2}$

54)  $h(x) = \frac{\sqrt{x^2-10x+25}}{x-5}$

55) Consider the function  $f(x) = \begin{cases} x^2 + kx & x \leq 5 \\ 5 \sin\left(\frac{\pi}{2}x\right) & x > 5 \end{cases}$

In order for the function to be continuous at  $x = 5$ , the value of  $k$  must be...  
(2 pts)

56) Consider the function  $f(x) = \sqrt{x-2}$ . On what intervals are the hypotheses of the Mean Value Theorem satisfied? (2 pts)

57) Find all critical values, intervals of increasing and decreasing, any local extrema, points of inflection, and all intervals where the graph is concave up and concave down. Show all work. (2)

$$f(x) = \frac{5 - 4x + 4x^2 - x^3}{x - 2}$$



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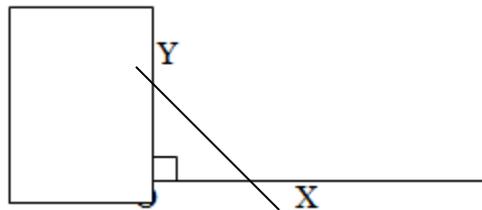
58)



The balloon shown is in the shape of a cylinder with hemispherical ends of the same radius as that of the cylinder. The balloon is being inflated at the rate of  $261\pi$  cubic centimeters per minute. At the instant the radius of the cylinder is 3 centimeters, the volume of the balloon is  $144\pi$  cubic centimeters and the radius of the cylinder is increasing at the rate of 2 centimeters per minute. (The volume of a cylinder with radius  $r$  and height  $h$  is  $\pi r^2 h$ , and the volume of a sphere with radius  $r$  is  $\frac{4}{3}\pi r^3$ .)

- (a) At this instant, what is the height of the cylinder? (2)
- (b) At this instant, how fast is the height of the cylinder increasing? (2)

59)



A ladder 15 feet long is leaning against a building so that end X is on level ground and end Y is on the wall as shown in the figure. X is moved away from the building at a constant rate of  $\frac{1}{2}$  foot per second.

- (a) Find the rate in feet per second at which the length OY is changing when X is 9 feet from the building. (2)
- (b) Find the rate of change in square feet per second of the area of triangle XOY when X is 9 feet from the building. (2)



#60-66 Integrate. 2 pts each

60)  $\int_{-8}^{-1} \frac{x-x^2}{2\sqrt[3]{x}} dx$

61)  $\int_{-\pi/6}^{\pi/6} \sec^2 x dx$

62)  $\frac{d}{dx} \int_1^x \sqrt[4]{t} dt$

63)  $\frac{d}{dx} \int_{\sin(4x)}^0 e^t dt$

64)  $\int \frac{x^3}{\sqrt{1+x^4}} dx$

65)  $\int \frac{\csc^2 x}{\cot^3 x} dx$

66)  $\int \sqrt{\tan x} \sec^2 x dx$

67) What are all the values of  $k$  for which  $\int_2^k x^5 dx = 0$ ? (2)

68) The function  $f$  is continuous on the closed interval  $[1, 9]$  and has the values given in the table. Using the subintervals  $[1, 3]$ ,  $[3, 6]$ , and  $[6, 9]$ , what is the value of the trapezoidal approximation of

$x$	1	3	6	9
$f(x)$	15	25	40	30

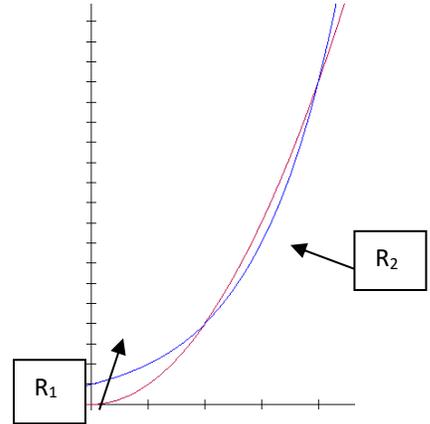
$\int_1^9 f(x) dx$ ?

Use a right Riemann sum with 5 subdivisions to approximate the area under the curve of  $y = h(x)$  on the interval  $[0, 10]$ . (2)



69) The shaded regions,  $R_1$  and  $R_2$  shown above are enclosed by the graphs of  $f(x) = x^2$  and  $g(x) = 2^x$ .

- Find the x- and y-coordinates of the three points of intersection of the graphs of  $f$  and  $g$ . (2)
- Without using absolute value, set up an expression involving one or more integrals that gives the total area enclosed by the graphs of  $f$  and  $g$ . Do not evaluate. (2)
- Without using absolute value, set up an expression involving one or more integrals that gives the volume of the solid generated by revolving the region  $R_1$  about the line  $y = 5$ . Do not evaluate. (2)



70) Let  $R$  be the region in the first quadrant under the graph of  $y = \frac{1}{\sqrt{x}}$  for  $4 \leq x \leq 9$ .

- Find the area of  $R$ . (2)
- If the line  $x = k$  divides the region  $R$  into two regions of equal area, what is the value of  $k$ ? (2)
- Find the volume of the solid whose base is the region  $R$  and whose cross sections cut by planes perpendicular to the  $x$ -axis are squares. (2)